Structural Analysis as Driver in Surface-Based Design Approaches

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This research argues for novel strategies to integrate structural analysis data in architectural design. Instead of a linear procedure of analysis, synthesis, evaluation and post-rationalization a synthesis/evaluation loop is installed which embeds structural analysis data as design driver from early on. The approach regards structural performance as one design criteria within a network of different requirements. An equilibrium of multiple parameters is aspired to instead of a single-parameter-optimum.

The research is conducted via a custom-made digital interface between 3d modelling software and an application for structural analysis of space frames. The information exchange provides the basis for successive strategies within a collaborative design process of spatial roof structures: negotiation of an overall form and a multi-dimensional improvement of space frame topologies by a Genetic Algorithm (GA).

1. The protagonists of a collaborative design process

Designing buildings or structures can be described as a process comprising the analysis of a problem, the synthesis of a design proposal and the evaluation of this proposal. This 'trial and error' process is conducted in a feedback loop until one overall satisfying solution is developed.

This description is valid for the generative work of both, architects and engineers who share common processes and strategies when working in their specific realms. Solutions to structural and architectural design problems are not always obvious nor are the paths that lead to them. In consequence the analytical and the generative fractions of the work are not separate, consecutive tasks. As such linearity cannot account for the complexity of architectural projects. Additional knowledge gained through such iterative processes may require further analysis of the specific context or even the adjustment of previously defined design objectives [1]. David Billington describes structural design as an art independent from architecture. Engineers design huge structures with mono-functional requirements and shapes which are driven by the highest impacting force. The shape of the Eiffel tower for example resembles the moment distribution under horizontal forces. The goal is to reach a maximum efficiency in material use and construction costs while at the same time creating an aesthetic structure [2]. Architectural tasks with their diverse criteria can be understood as networks of interdependent nodes of requirements from which only a fraction is quantifiable. All nodes in this network like program, circulation, light, material, structure, budget etc. are interrelated and interact with each other. Building components never serve only one purpose and practices like UN Studio look for inclusive strategies that map a multitude of functions to single elements [3]. A separated optimization of single aspects within such a cluster of requirements as proposed by John Page [4] is not appropriate because it ignores the intensive interactivity between the single aspects. From this point of view, it is more suitable to understand design as a process of negotiation between problem and solution as described by Lawson [1], in which structural and economic efficiency of material use is just one, but not the only goal to reach. Thus the quality of a design proposal can be measured by the degree of equilibrium in the network. Faced with such a complex network of requirements or such a highly constrained situation, as Jane Drake [5] calls it, a design proposal is most probably not derived from a preceding analysis alone. The quality of a design cannot be anticipated in advance and not all given constraints can be considered in the first proposal. Interviews with architects conducted by Drake showed that designers rather tend to approach a complex problem by generating a relatively simple idea to narrow down the range of possible solutions and construct and analyze a first scheme accordingly. Lawson describes this process as

> "…first decide what you think might be an important aspect of the problem, develop a crude design on this basis and then examine it to see what else you can discover about the problem." [1]

In consequence Drake proposed to replace the analysis-synthesis-evaluation loop by a diagram comprised of generator-conjecture- analysis [5]. Conceiving design as a solution-focused process is reinforced by the common practice of architectural design competitions. Competitions invite architects to develop proposals based on detailed briefings which represent a major part of an analysis of the existing situation. Nevertheless every competition reveals as many solutions as there are participating architects. Even so, the quality of proposals varies: the range of solutions shows that the analysis is not yielding the optimum solution in itself.

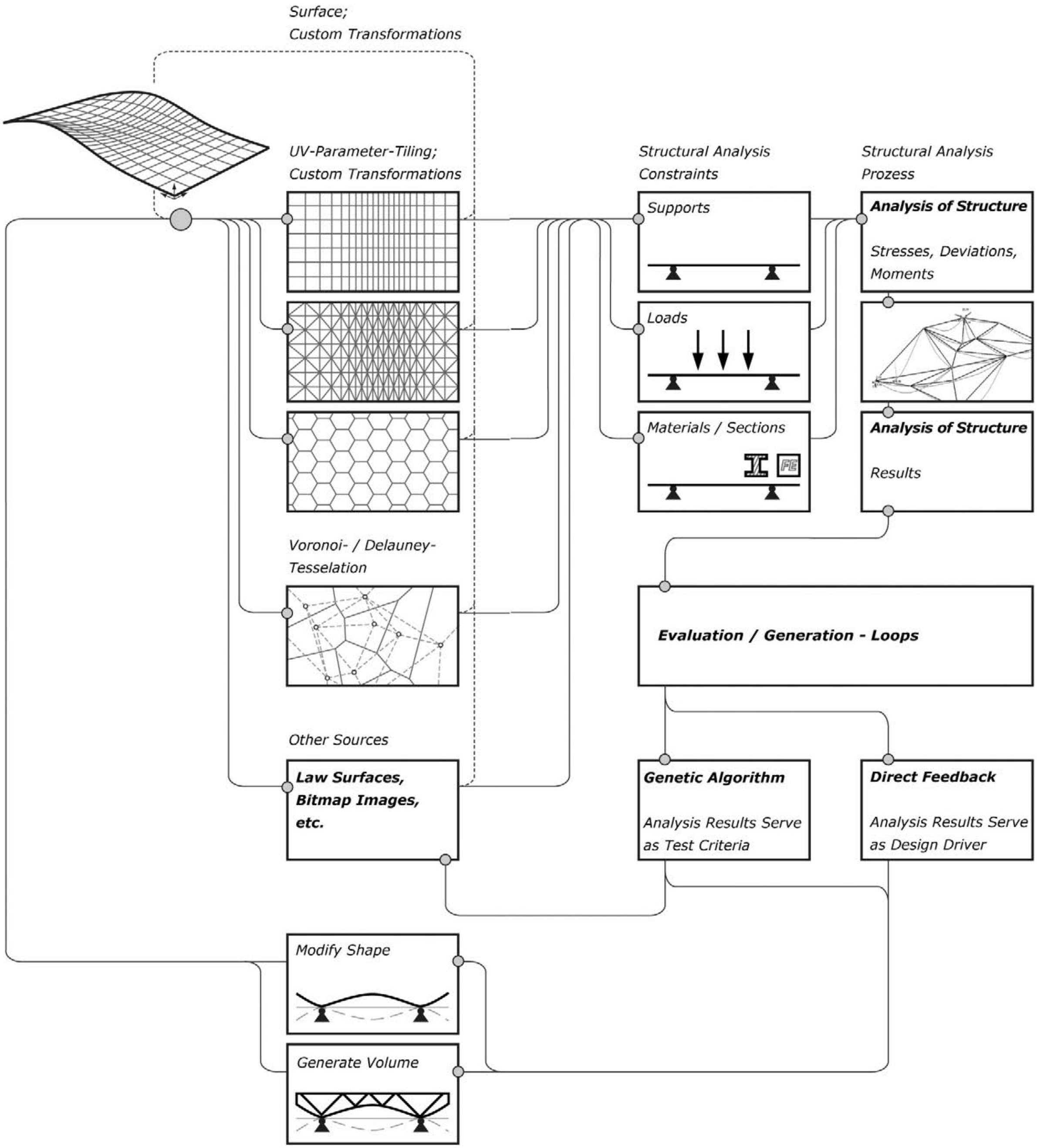
Against this background of commonalities and distinctions this research aims for a further integration of structural analysis in the generative part of a collaborative design approach. Various stages in the process are investigated below. At first well known form-finding strategies are transferred into the digital realm and applied to architecturally designed shapes to initialize a negotiation of architectural and structural demands. Second a GA is utilized to search a solution space that is spanned by structural and quantifiable architectural parameters.

2. Negotiate form

Surface-based design approaches have become widely used in architecture through B-Spline-modeling software. A surface can serve as a representation for a multitude of different aspects of a design beyond its initial function as a border between a volume and its environment. It is a two-dimensional object unfolded into the three-dimensional space without any material thickness. But instead of treating surfaces as border conditions, they are often perceived as the objects themselves. This misconception of a surface virtually representing a physical building component is – at the latest – revealed when the surface is translated into built form with structural requirements, material thickness and properties. To overcome this conceptional flaw and to accomplish construction requirements the strategy of the "*design surface* "[6] was recently introduced into the design practice. The two-dimensional surface is used as a host which is associated with secondary geometry, representing three-dimensional building components. These components fulfill fabrication constraints and at the same time represent the overall morphology of the initial surface. The concept of the design surface is used in this research to generate a mesh of structural elements along the guiding geometry of the initial surface. Bending flexibility of these members is used to improve the form of a surface structure and to activate its in-plane performance.

As shown in Figure 1 the developed design process starts with a surface that embodies an architectural design idea, which then is virtually driven and sequentially informed by a network of different constraints. One initial surface is translated into one or more custom meshes, which then are exposed to their dead load in the structural analysis software. The resulting deflections is fed back into an algorithm that steers the appropriate reactions towards the local stresses and deformations: The geometry of the

▼ Figure 1. Flow chart of the project



initial mesh is slightly altered to reduce bending, while still embodying the initial architectural proposal. Unlike in sequentially organised form-finding procedures, this process negotiates the form-driving 'forces' of both realms, the structural and the architectural. The new, altered mesh then is transformed back into either a single surface or a two-layer surface model in the 3d-modeling software. In the latter case the in-between space now describes a volume for space frame construction. From this new starting point a Genetic Algorithm is used to find a solution of a space frame which again deals with different architectural, structural and economical constraints. In the example shown below, these three factors are represented by spaces which should be free of structural elements, a minimal number of structural members, and a minimum of over-all deviation. The structural analysis data, which result from that next feedback loops, finally are used to generate in a simplified, but parametrically driven manner, all structural members of the developed space frame system. As the flow chart of the project indicates, this is one way amongst others to work with the developed tools.

The project uses a 3d-NURBS-Modeler (Rhinoceros™) for surface generation, surface modification and the visualization of geometry. Structural analysis is done in the therefore specialized application RSTAB™. These two generic tools are tight together by some custom made programs (in Visual Basic for Applications™), which are basically used as data interface for automated tasks such as different ways of surface tessellation. Additionally a Genetic Algorithm which is generating and evaluating data in Rhinoceros™ and RSTAB™ was developed for this project.

2.1 From surface to mesh

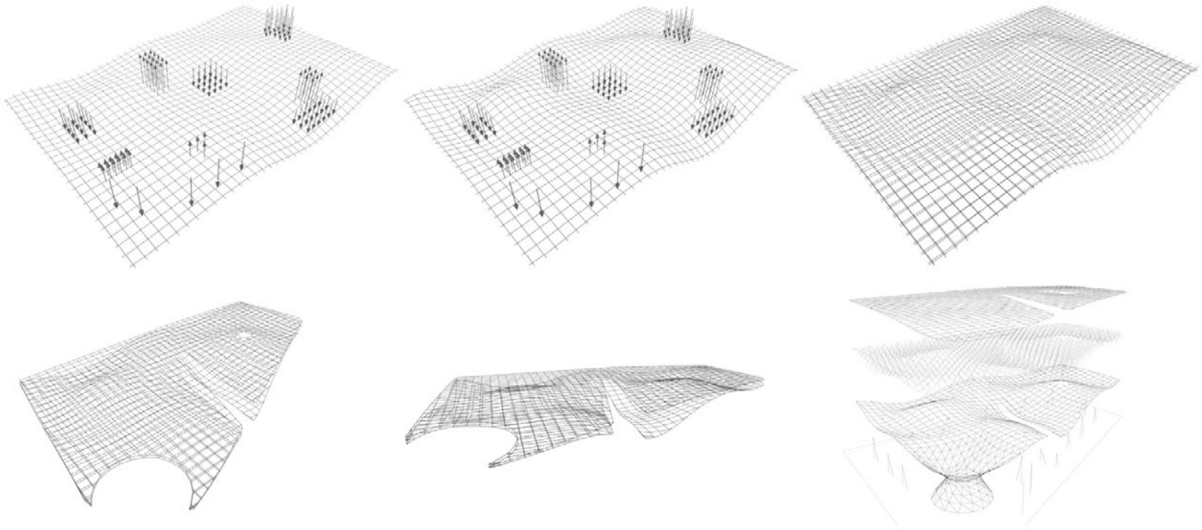
The uv parameter space of the surface is used to generate meshes with specific topologies along the surface. Triangular, quadrangular or hexagonal meshes with different resolutions can be generated depending on the designer's choice. The mesh incorporates the overall shape of the initial B-spline surface.

Before transferring the mesh into the analysis software the supports have to be defined. Supports are located at the nodes of the mesh. Their location can be defined in different ways: the position and number of supports can be chosen deliberately, they can be placed randomly, at the corners or at the lowest nodes of the mesh. Mesh and support positions are transferred into the structural analysis software for space frame structures. The architectural language represented by the B-spline surface is transcribed into a set of data suitable for structural analysis. Nevertheless in this stage of the process it is important to stress that the analysis software is considered as a kind of form finding device and not as a tool for structural analysis.

2.2 Form improvement

The mesh does not act as a proper structural system yet. But exposed to its dead load its nodal deviation shows similar behavior to a hanging chain model. Depending on the topology of the mesh, the bending resistance of its members and the support positions, the initial mesh is deformed. Turning the deformation upside down leads to shell-like structures with reduced bending moments. This strategy is well known from the hanging models of Frei Otto. The grid shell of the Multihalle in Mannheim by Buro Happold and Manfred Mutschler with Frei Otto is an example for such a form-finding process. The 3-D curvature of the shell is derived from a non-rigid net that constitutes in a spatially curved form when suspended from peripheral support points. Turning the 'frozen' net upside down leads to a form which is free from moments and stressed by compression in axial direction. The use of minimum bending stiffness of structural members was pursued in the construction [7]. Kilian transfers the concept of the hanging model into the digital realm. His system provides a real-time three-dimensional-modeling environment that mimics hanging chain behavior through a particle spring system. Furthermore a technique is proposed to instantiate chain segments by structural elements whose dimensions are driven by present forces [6]. Form-finding is appreciated in this research as a generative means. However, the difference lies in the aim to integrate a wider range of form-driving parameters into the design process. An architectural project that exemplifies this approach is the BMW Welt by COOP HIMMELB(L)AU together with the engineers Bollinger + Grohmann. The complex roof structure of the building was designed in a collaborative process. During the competition a double layered girder grid was developed by the engineers which demarcates the upper and lower boundaries of the roof space phase in alignment with the architectural concept of a floating cloud. Driven by the simulation of anticipated loading scenarios the initially planar girder grid was deformed such that the upper layer assumed a cushion like bulge as shown in "Figure 2". The lower layer also reacts to a number of spatial and structural criteria. For example, the roof integrates the customer lounge, a large incision that opens the views towards the famous BMW headquarter tower and channels the forces to the defined bearing points. The combined capacity of both girder grid layers to act as one spatial structure with locally differentiated behavior is achieved through the insertion of diagonal struts within the interstitial space. In response to local stress concentrations the structural depth of the system varies between a maximum of twelve meters and just two meters in areas of less force.

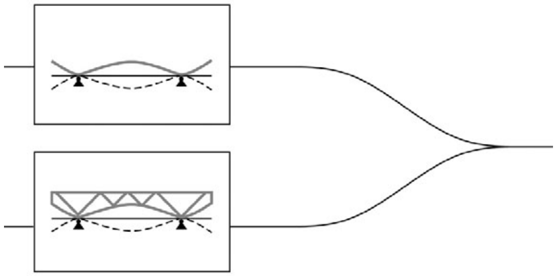
In the northern part of the building the roof merges with a double cone, typical for COOP HIMMELB(L)AU's work, to form a hybrid shape. Similarly, the related bending behavior of the roof structure gradually transforms into the shell like behavior of the double cone [8]. Inspired by the collaborative approach of architects and engineers the start condition of the form



negotiation process is defined by a B-spline surface incorporating a morphology that is driven by architectural criteria not necessarily related to structural issue. It is not a neutral plane that is transferred into a manifestation of the existing force flow but an articulated and differentiated shape that is exposed to another parameter in the design process: the structural performance.

▲ Figure 2. Stills of the BMW Welt roof form-generation process. (Copyright: Bollinger + Grohmann GmbH)

The analyzed nodal deviation is fed back into an algorithm that steers the appropriate reaction towards the local stresses and deviations, taking into account the mesh topology, its supports and their position in the mesh. The interpretation of nodal deviation is followed by two approaches to integrate these results in the architectural design process. Both approaches are currently investigated.



◀ Figure 3. Form improvement and structural volume.

2.3 The single-surface form-improvement

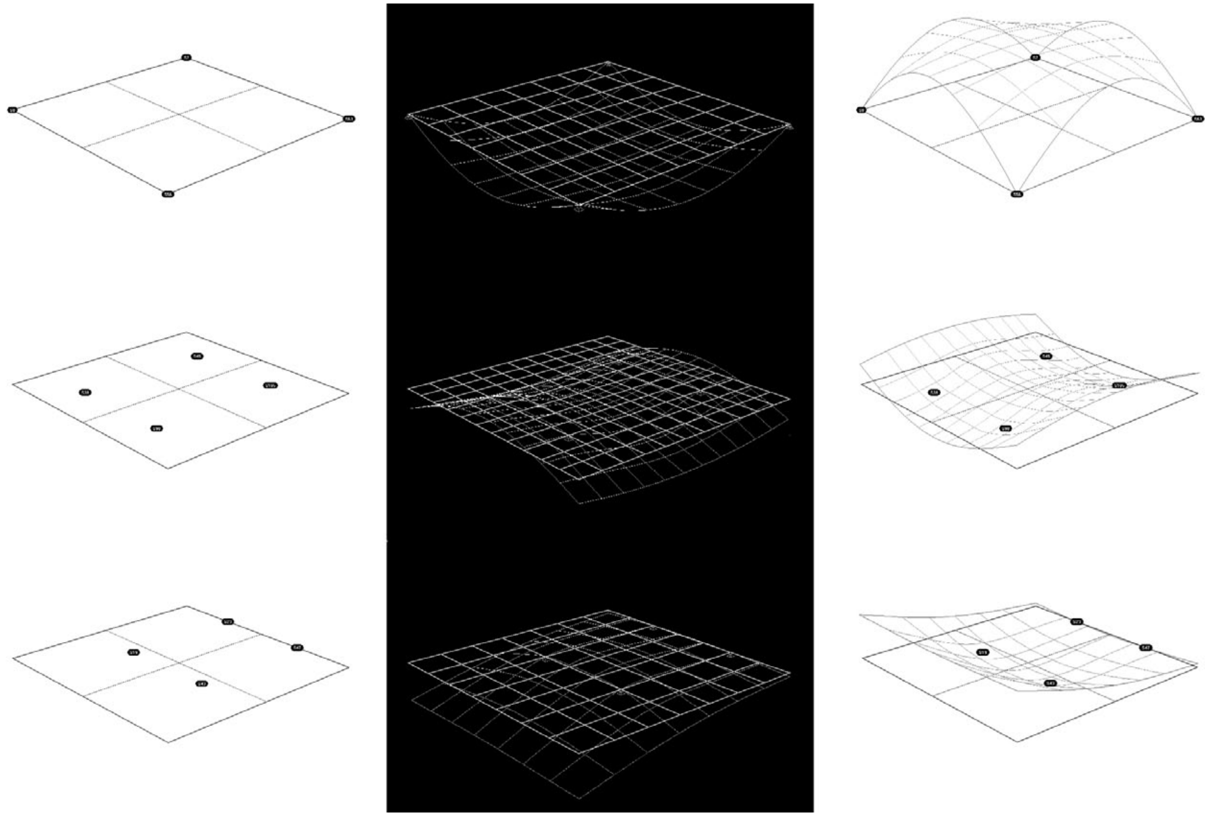
The elegant shells of Felix Candela embody the structurally perfect and ideal shapes. Ove Arups assumed the reason for the excellence of Candela's works was due to him combining architect, engineer and contractor in one person, with a clear dominance of engineering over architecture. He believed that the creative process needs to be synthesised in one mind, which is aware of all aspects relevant to a project's success [9]. As this situation is extraordinarily rare architecture usually is a product of a collaborative effort. Thus even a shell has to integrate a wide range of design criteria far beyond structural aspects only. From an engineering perspective, homogeneous, idealized shells are elegant as they transfer forces without incurring bending forces and thus can be constructed with minimal material thickness. However, any incision in such an 'ideal' shape, as for instance a door, leads to fundamental, problematic changes in the structural behaviour. SANAA's "Learning Center" project for the campus of Ecole Polytechnique Fédérale Lausanne (EPFL) for example is an undulating landscape building which includes patios, openings and various spatial qualities and thus results from a design process, in which structural aspects were just one set of design criteria amongst many.

▲ Figure 4. SANAA's "Learning Center" , Lausanne (Copyright: Kazuyo Sejima + Ryue Nishizawa/SANAA)

Rather than prioritizing idealized geometries as known from Candela's projects, here the work focused on analysing every node according to its relative position in the mesh and to the supports. Areas between supports behave like a hanging model that constitutes in a spatially curved form when suspended from support points. Reversing this deformation creates shell or arch behaviour of single or double curved grids with forces increasingly bundled in tension and compression. Surface and grid structures like shells or domes resist forces through their double-curved form and integrity. The bearing mechanism is achieved by a membrane-like behavior. Like a balloon that is not able to resist bending moments with its thin surface shells resist external loads through tension. In case of symmetrical loads the form will be kept in equilibrium by meridional forces and ring forces only. This state of pure membrane-tension can only be achieved by a hemisphere with continuous linear support, a structurally ideal condition which is rarely desired in practice as openings and punctual supports are necessary for programmatic issues. Therefore the presented approach seeks to generate local areas of shell or arch behaviour, without neglecting the architectural shape. 'Classic' form finding is superseded by processes of tracing and generating performative capacities in the specific morphology. As the load bearing characteristics vary across the shape, no region represents a pure structural typology or a structurally optimized shape.

Cantilevering parts of the mesh are treated differently than nodes between supports.

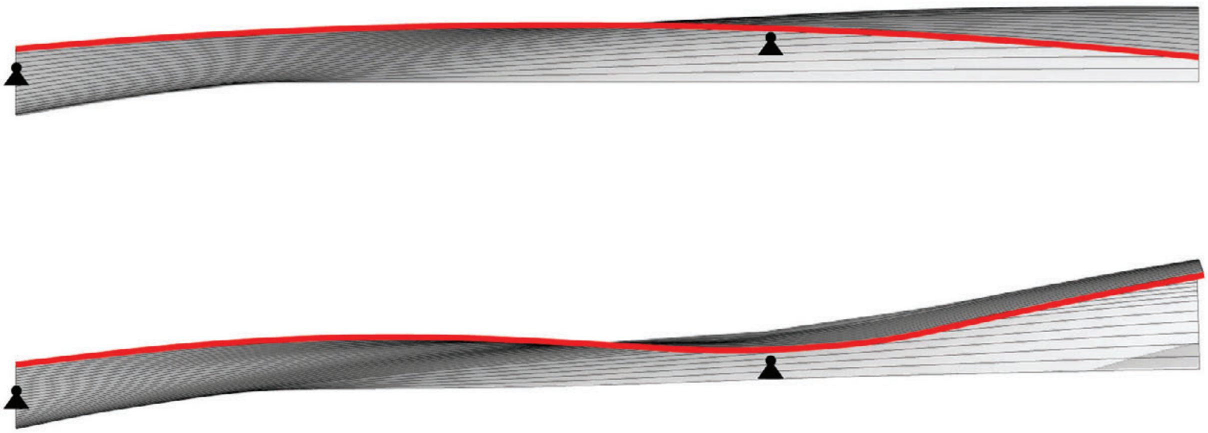
These areas have both types of forces within the same member. Reversing the nodal deviation would not be a suitable way to deal with cantilevers. A geometric improvement in general is less effective in these areas. However, cantilevering structures are consciously included in this research because it is a common element in architectural design practise.



▲ Figure 5. Three initial surface/support conditions and the resulting surfaces exemplify the process

The deviation of cantilevering nodes is set in relation to the maximum deviation so that affected parts are lifted slightly as exemplified in Figure 5.

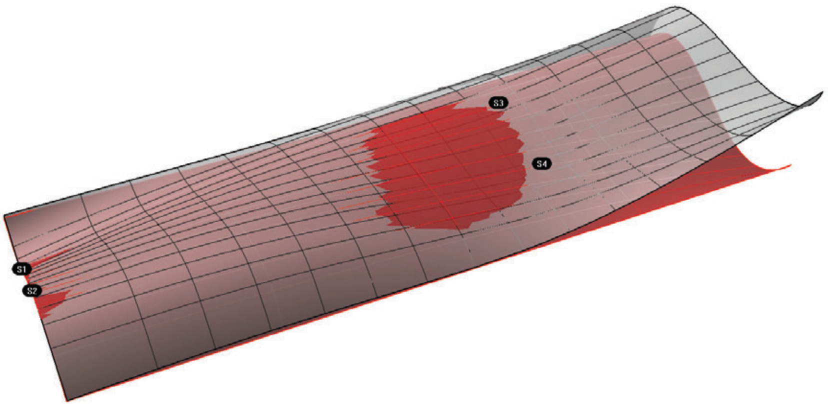
The refined geometry leads to a structure with increasing axial forces. Since the initial surface is not necessarily symmetric and the supports are located in inappropriate positions axial forces will always be accompanied by bending. The form is not "found" by the most efficient force flow but instead the already articulated geometry is informed and differentiated by use of evaluation data.

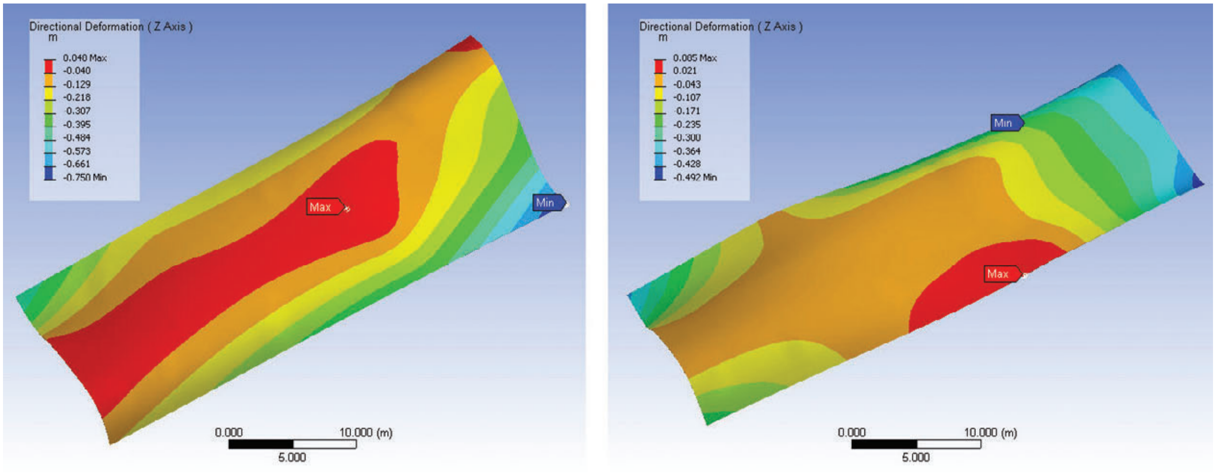


▲ Figure 6. Section of improved (top) and non-improved (bottom) form

The result is a mesh that is fed back in the 3D modelling software based on the improved nodal positions. This approach was tested with a surface that went through the above described process. The optimised mesh is transferred back into a B-spline surface. The derived surface is supposed to incorporate an improved structural action concerning deformation because its overall shape is informed by structural analysis data. To prove this assumption the initial surface and the informed surface are compared in a finite-element analysis software. The result shows a significant decrease in deformation in the cantilevering area which also leads to less bulging between the supports, as shown in Figure 8.

► Figure 7. Non-improved (red) form, superimposed improved form (grey)



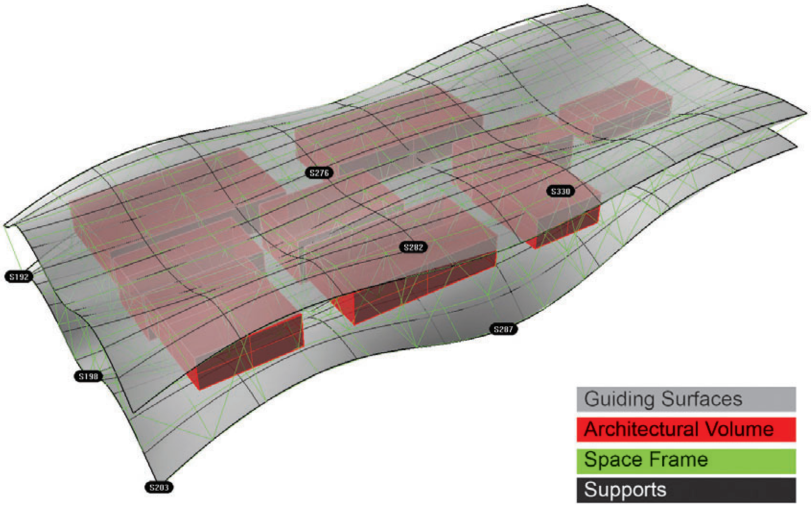


▲ Figure 8. FEM analysis of non-improved (left) and improved (right) form

The improved surfaces still incorporate the morphology of the initial surface which was driven by architectural design parameter. At the same time the shape is slightly deformed and adapted to inherent stresses and deviations. Structural analysis is becoming a driver for geometry [10]. Differential mesh topologies, mesh resolutions and support positions and their influence on the resulting geometry are tested in several iterations. The interactions of surface geometry and mesh topology can be analysed instantly through the provided interface between both software packages. An instant feedback from generation to analysis and vice versa is installed.

2.4 Double surface model

In the second approach an additional layer is added. The initial mesh evolves into a double-surface model in the 3d software. The distance between both surfaces refers to the local deflections in the evaluated mesh. The surfaces now act as the border representation of a volume and not as objects



◀ Figure 9. 3D Model for evolving space frames themselves. The conceptional flaw of surfaces with zero thickness is overcome by a volume that can be filled with a space frame between upper and lower surface. The system is generated by nodes and interconnecting poles.

3. EVOLVING SPACE FRAMES

Vector-active lattice systems like space frames are very common in structural design and can be traced back to an innovation in structural analysis in 1866. Karl Culmann, Professor of engineering science at the 'Eidgenössischen Polytechnikum' in Zurich, published his 'Graphic Statics', including his developments of the most important graphic methods for calculating structural behaviour[11]. With these methods being particularly appropriate to calculate lattice girders no other structural typology signifies better the succinct impact of new calculation methods on the changing understanding and employment of structures [12]. Not until the 1960's the graphic statics were replaced by computer numeric procedures whereas the generic structural typologies established by the former scientific methods remained the dominant systems.

In this research the space frame becomes the next objective of multi-dimensional negotiations. Instead of utilizing a generic structural type whose parts are subsequently evaluated the entire system is improved. Improvement is regarded as the balance of multiple architectural and structural requirements. The architectural requirements are represented by the double layer surface system that describes the desired overall morphology of the roof. In addition volumes are defined between the surfaces that should be free from structures to provide usable spaces in an architectural sense.

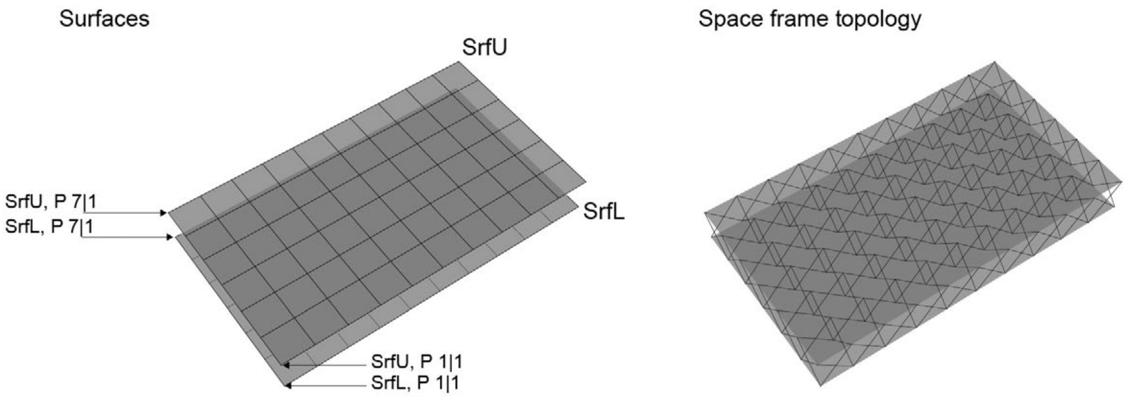
The initial space frame topology (element-node connections) refers to conventional structural systems. The goal of structural improvement lies in the reduction of overall deflection of the system and the use of a minimal number of elements. This objective is assumed to be reached by locally differentiated behaviour of the system; double curved areas can generate shell-like behaviour which gradually transforms into bending behaviour in more planar areas of the system. The space frame is set up as a parametric model whose topology can be altered through external parameter.

Thus the boundary conditions are known and the desired goal is clearly defined. The parametric space frame model allows externally driven variation to achieve a solution space instead of a single solution. An adequate means to generate and navigate such a solution space is an Evolutionary Algorithm (EA). Those algorithms are based on mimicry of the process of evolution, which leads to the effect, that through the accumulation of small improvements over time the maintained solutions gradually converge towards the defined criteria. More technically speaking, there are four major types of Evolutionary Algorithms, but which all follow a common architecture, as described by Bentley [13]. In this case study, a custom made Genetic Algorithm (GA) is used, based on roulette wheel selection, the genetic operators of mutation and crossover and some elements of island injection, as introduced by Eby et al [14]. GA's are a common means for optimization problems in structural design and increasingly emerge in the field of architecture as a generative tool [15].

The GA inherits some touch of unpredictability, which makes it highly interesting for design processes. When using a GA, the elements of a design model and their properties are defined. But transformation processes, which are performed on the model, do not need to be rolled out in each detail. Of course it is necessary to determine *what* may be changed and a general rule how this may happen. But how it happens in detail, is strongly influenced by the random processes of mutation and crossover.

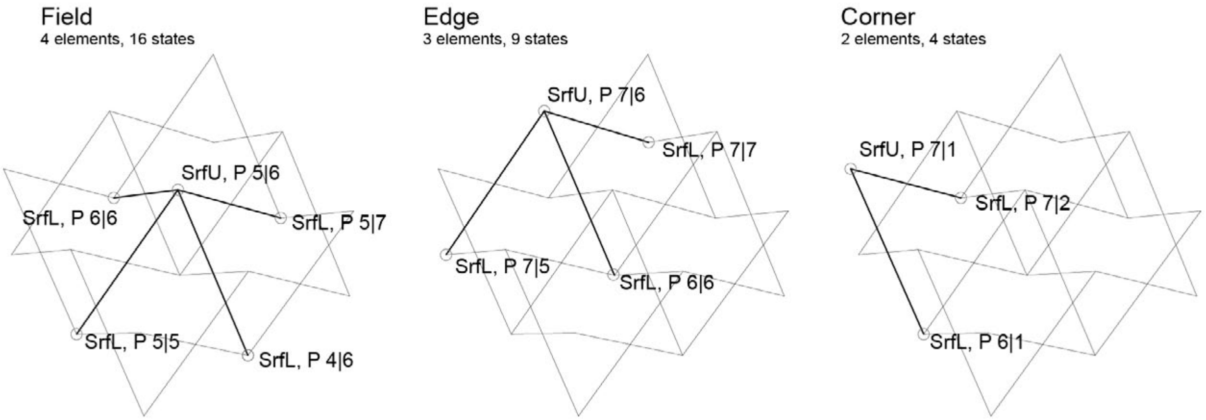
3.1 Model representation – phenotype and genotype

In the terminology of Genetic Algorithms one solution is refered to as an individual; a group of such individuals is called population. Each Genetic Algorithm proliferates a series of different solutions wich increase their performance over time. The improving quality is organized in subsequent generations. Better individuals have a higher probability of being chosen to generate offspring – new individuals, in this case new space frame systems. One individual consist of two parts: the phenotype and the genotype. The phenotype can be understood as one complete representation of an individual digital design model. The genotype is a generic, in this case binary, representation of individuals. In this research the diagonal elements between the two horizontal meshes are encoded in an array of bits. The environment of the GA is provided by the double-layer surface system. Both surfaces are translated into meshes with similar sample rates along their UV parameter space. Supports can be defined at any node of the meshes in response to the actual design task. The meshing procedure and support definition are not objectives of variation but defined in advance.



▼ Figure 10. Surfaces, meshes and space frame topology

The phenotype is represented by the diagonal elements that connect both layers to form a coherent space frame. Diagonal elements start from a node in the upper mesh and end at a node of the lower mesh that is in front, behind, left of right from the starting node. A node inside the field has four possible elements, a node at an edge has three possible elements and in a corner a node has two possible elements.

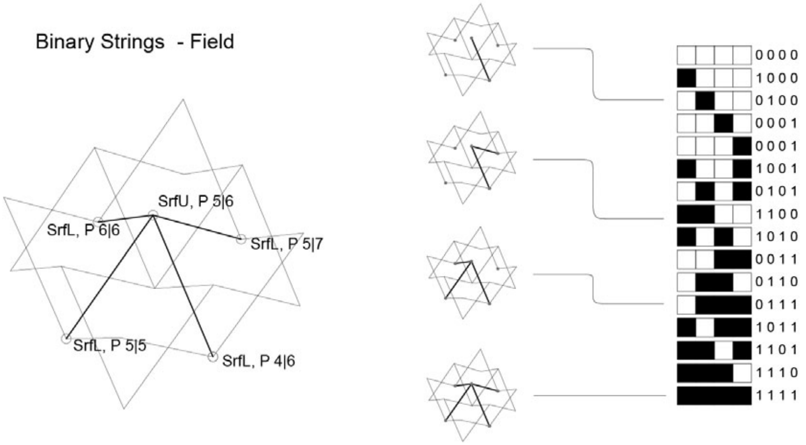


▲ Figure 11. Changing conditions within the space frame

3.2 The genome

The actual number of diagonal elements at a node is controlled by a binary genome. Every node of the upper mesh has two, three or four bits that represent their possible elements depending on the position in the mesh. A zero in the genome stands for 'no element' while a one stands for 'element'.

► Figure 12: The binary genotype controls the space frame topology



3.3 The Evaluation/Fitness Functions

3.3.1 Structural performance

Each space frame individual is evaluated and ranked by three fitness functions. The first fitness function creates a three-dimensional model in the structural analysis software RSTAB based on the information of the individual genotype. A cross section profile and a material are assigned to each element although this cannot be regarded as a proper dimensioning of the space frame. Of major interest at this moment is a comparison of deflection of the different individuals under the influence of dead load. The maximal nodal deflection is identified and the aspired deflection value of the fitness function is subtracted. This value quantifies the optimality of the solution.

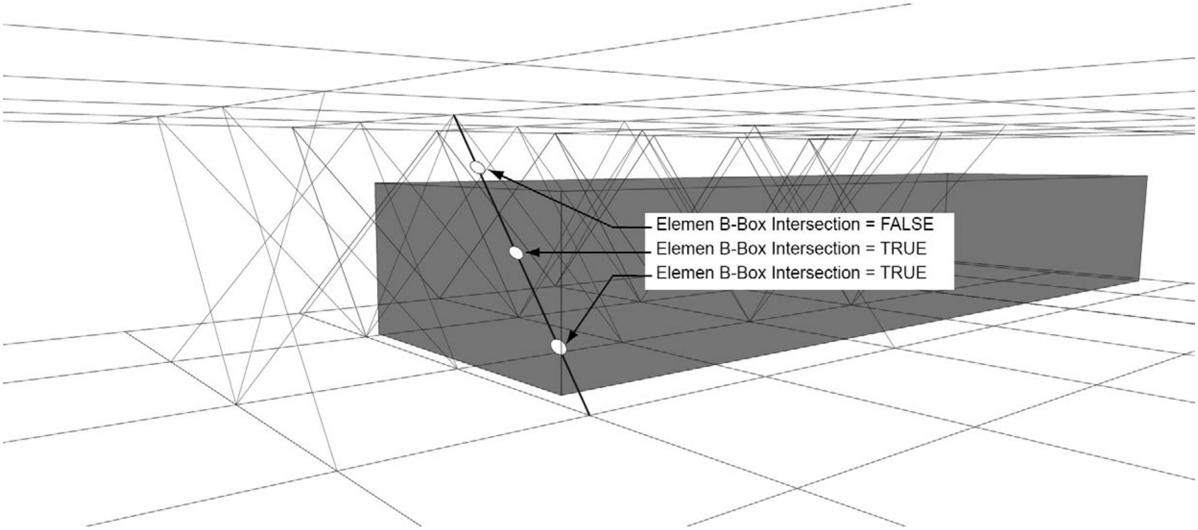
3.3.2 Number of elements

The second fitness functions simply counts the number of diagonal elements that incarnate in the phenotype. The fewer elements an individual presents the higher it climbs in ranking. This fitness function obviously constitutes a conflicting interest to the first fitness function which ranks rigid structures higher than those which show significant deformations.

3.3.3 Spaces without structure

The last fitness function checks whether an element penetrates defined volumes between the two horizontal surfaces. Three vertices along the element are analysed regarding their position in relation to the bounding boxes of the volumes that should be free from structural elements for architectural reasons. The fewer points lie inside the bounding boxes the higher the individual is ranked. The volumes provoke singularities which have to be incorporated into the system.

▼ Figure 13. The third fitness function checks how many elements intersect with the volumes that should contain no structure



3.4 The evolutionary progress

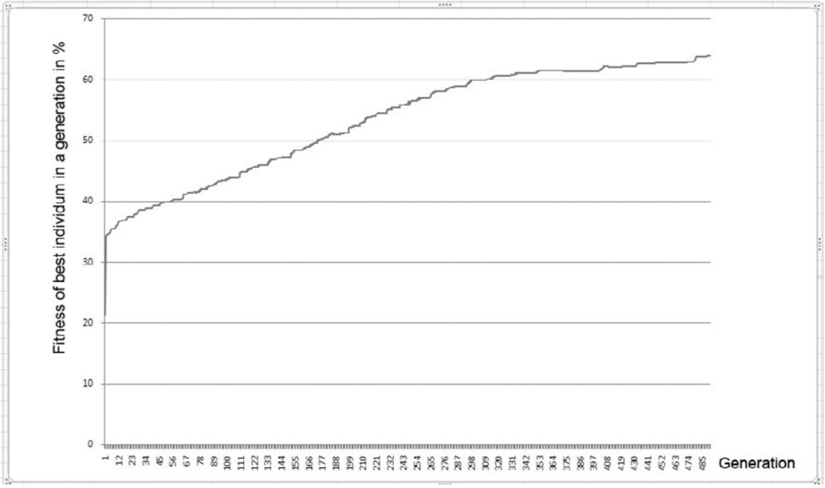
After all individuals of all populations are generated and ranked by the fitness functions the space frames are selected by the roulette wheel method. This stochastic selection procedure is performed on basis of the fitness values of each individual and can be understood as a rotating roulette wheel with differently sized sectors for each fitness value – better fitness values get more, worse ones less space on the roulette wheel. Thus better ranked space frame individuals will survive more likely but also weaker individuals are not completely without chance. This selection procedure prevents from early stagnation in the development because even weaker individuals may inherit properties that might prove successful in future generations.

The individuals selected by this procedure are used to produce offspring for the next generation. The natural process of recombining chromosomes by mating is emulated by the GA through the genetic operator crossover. The genetic material is passed to the next generation whilst simultaneously recombined. In this project the two bit strings representing two genotypes are split at one random position and the resulting sections are swapped to recombine one new genotype.

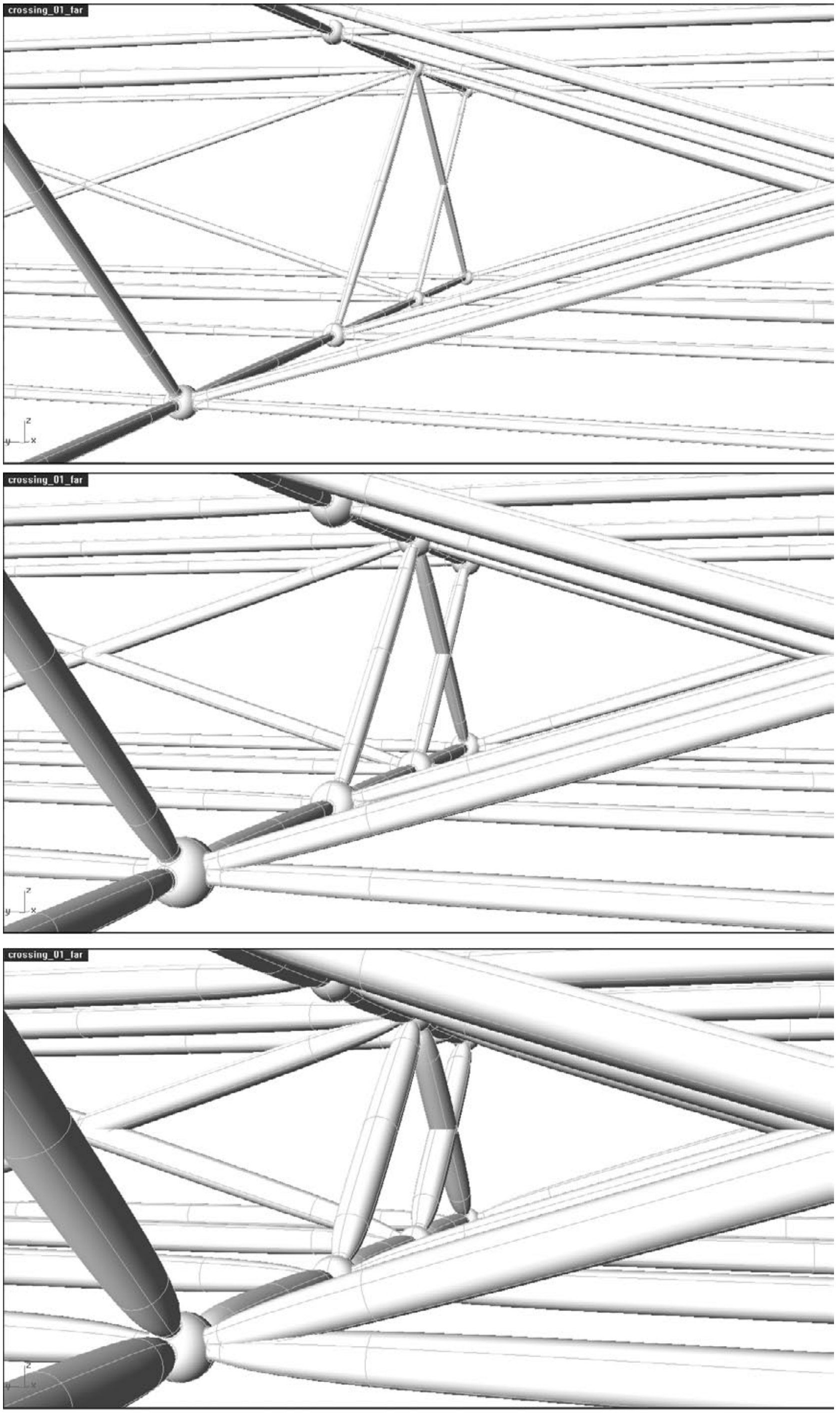
The above described process is accompanied by another means to diversify the chromosome pattern: Mutation is implemented in the GA by randomly flipping a bit in the genotype. A '1' becomes a '0' and vice versa. The two genetic operator mutation and crossover vary the number and position of the diagonal elements connecting the upper and lower part of the space frame mesh.

3.5 Results

The evolved space frame shows an improved performance regarding a significant decrease in element numbers while maintain only little deflection.



► Figure 14. Representation of the evolution of the fittest individual in every generation



◀ Figure 15. Three different dimension scenarios for the same center-line model.

The process starts with a high number of elements (193 in generation 1) generating a rigid space frame with small deflection of 55mm. During the evolution the number of elements decreases (89 in generation 500) while the deflection is not significantly changing. The number of elements penetrating the architectural space is reduced from 193 in generation 1 to 69 generation 500. The graph in Figure 14 shows the potential for a further improvement through a higher number of generations.

A centre line model represents the resulting space frame solution. This generic data has to be instantiated by a material system, a subsequent task which is only briefly addressed in this paper by a system of differentiated elements that merge with spherical nodes. This approach depicts one possible way to further interpret structural analysis data. The space frame went through another structural evaluation which returned the individual internal forces for every element. This data was used to control the differentiated dimensioning of the space frame as shown in Figure 15. This data set could be augmented by additional parameters like material properties, codes and other aspects of a specific project, an issue which exceeds the current research. The tools developed so far are regarded as rather generic and therefore deployable in a wide range of tasks. Instantiation of the center-line-model by geometry which represents material systems has to happen in a specific project.

4. Conclusions

Integrating the generative means of form-finding into a design process with a broader range of parameters proved successful. Altering local areas of the global geometry while simultaneously maintain the architectural expression led to less deformation in the shape. At the same time the formal repertoire is not limited to shapes that are entirely force-driven. Being able to automatically transfer the data from a surface model to a structural mesh and back again provides a fast and easy way to augment the mere geometric architectural model. The design process is driven by architectural and structural parameters simultaneously.

The negotiation between different and even conflicting aspects yields solutions which are improved during the generative process instead of being post-rationalised afterwards. The procedure is seen as an approximation for the early design stage that offers architects the possibility to integrate structural aspects into their work and use them as design driver.

The use of genetic algorithms within the design process is a powerful method to generate variation and at the same time search a solution space. Especially when searching for a balance of multiple or even conflicting interests this heuristic approach is beneficial. Evolving quantitative factors by digital means is a congenial partner of human generative capacity as long as both complement one another. Such task sharing in the man-machine ‘partnership’ is regarded by the authors as the most promising approach.

Jane Drake described the primary generator diagram [5] comprising of generator-conjuncture-analysis which proposed to start with a rough initial design which narrows the solution space could be instantiated by a GA that acts as a generative means. Improve analysis by synthesis is enforced by the multitude of possible solutions which are produced randomly in the beginning and refined during the process. By using a GA, variation is not driven by a specific goal but happens stochastically. The selection constitutes the only control mechanism to direct the development. As in nature, individual fitness is evaluated on the phenotypic level (in this particular case: a 3d model is generated an evaluated) as the likeliness for further reproduction [16].

The collaborative design process could be improved by evaluating structural and architectural criteria simultaneously. The equally ranked requirements of maintaining the overall morphology, free span intermediate spaces and minimal deflection of the space frame could be satisfied.

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